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SHORT NOTE

Variable-heave models of deformation above listric normal faults: the importance of area conservation

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Abstract—In recent years work on extensional fault-controlled structures has illustrated the relevance of spatial heave variations along faults in producing structures such as hangingwall synclines. In this note I mention two models which have been described: in one, the fault heave varies and vertical marker lines remain vertical (the 'modified Chevron model'), whilst in the other, markers perpendicular to the fault remain perpendicular during movement (the 'slip-line model'). I show that both these models produce large area changes and cannot be used quantitatively unless these spurious effects are removed. I give here the derivation of models which satisfy the same general criteria but conserve area. Some aspects of the amended models are detailed. Both the modified Chevron model and slip-line model predict strains which increase without limit away from the fault plane, and in the slip-line model infinite strains may be predicted in geologically reasonable scenarios.

INTRODUCTION

INTEREST in the effects of deformation above listric normal faults was stimulated by the constant-heave model presented by Verrall (1981). Since then various new models for hangingwall deformation above extensional faults have been derived (e.g. Coward & Gibbs 1986, Williams & Vann 1987). These models attempt to include the effects of varying displacement along faults and are thus a generalization of the constant-heave (vertical simple shear) model. The models are used to generate structures similar to those sometimes imaged on seismic sections, such as hangingwall synclines above gently curving faults.

One of these new models (the 'modified Chevron' of Williams & Vann 1987) allows heave along a fault to vary while preserving the heights and orientation of vertical lines in the hangingwall. The other model discussed by these authors (the 'slip-line model') specifies that lines perpendicular to the fault remain perpendicular, and retain their lengths, as they move down the fault. The purpose of this paper is to show that these two models produce gross area increases and decreases during hangingwall deformation and therefore in their given form are of qualitative use only. In this discussion I derive amended versions of the models, in which area is locally conserved whilst still allowing heave to vary. I then state some limitations to the two models which should be borne in mind whenever they are used.

VARIABLE-HEAVE MODEL

In this model vertical lines remain vertical, but may move together or apart, as the hangingwall moves over a fault surface. These vertical lines retain their height during the deformation. However, if such lines change their relative spacing then the area of the slab of rock which they bound will increase or decrease. This is a spurious effect of the Williams & Vann (1987) model. In reality the changing separation of vertical lines will be accompanied by changing height; thus if the heave gradient doubles the spacing of vertical lines, it follows that their heights should be reduced by 50%. In a feasible model, let X represent the x co-ordinate of a vertical marker line before deformation and let x be its final position

$$h = x - X.$$

If the heave (h) changes, then the spacing of vertical lines changes from S to s (Fig. 1) according to

$$s/S = \mathrm{d}x/\mathrm{d}X = 1 + \mathrm{d}h/\mathrm{d}X.$$

Thus if the heave decreases with X, the hangingwall shortens and $s/S < 1$. To conserve area everywhere in the hangingwall, let Z and z be the initial and final heights of a given column of rock. Then

$$sz = SZ$$

so

$$z = \frac{Z}{1 + \mathrm{d}h/\mathrm{d}X}. \tag{1}$$

To illustrate this, consider a buried extensional fault tip (Fig. 1) at which the heave $h = 0$ and suppose the heave has value $h = 1$ at $X = 1$ (so $x = 2$). Then, the simplest hypothesis is that the strain is homogeneous in the extended zone. The strained zone must be bounded by vertical faults (Fig. 1). Such vertical faults will always appear in the modified Chevron model, bounding zones

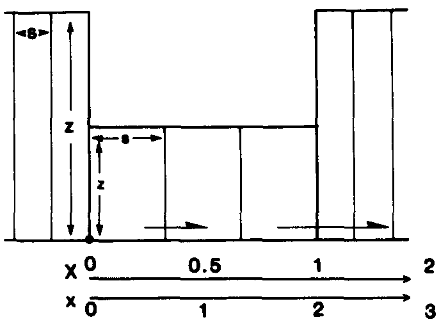

Fig. 1. Simple model of a buried extensional fault tip. S and s are initial and final spacing of notional vertical lines; Z and z are initial and final heights of rock columns. Dot marks fault tip, to left of which there is no slip.

in the hangingwall with differing heave gradients. More realistically the heave gradient dh/dX should itself be a continuous function of X, in which case vertical faults are not necessary. In the above example we may put $dh/dX = 0$ at either end of the extended zone. As an example, a simple function satisfying these criteria is a trigonometric function

$$dh/dX = 1 - \cos(2\pi X)$$

so

$$z = \frac{Z}{2 - \cos(2\pi X)}$$

and

$$h = X - (1/2\pi)\sin(2\pi X). \quad (2)$$

The result of this is shown in Fig. 2. The important implications of this model are that in addition to the depth increase due to moving down an extensional fault, a depth increase proportional to the thickness of the hangingwall is imposed. These significant strains are a consequence of area conservation.

SLIP-LINE MODEL

The essence of this model is that straight lines perpendicular to the fault in the hangingwall remain straight

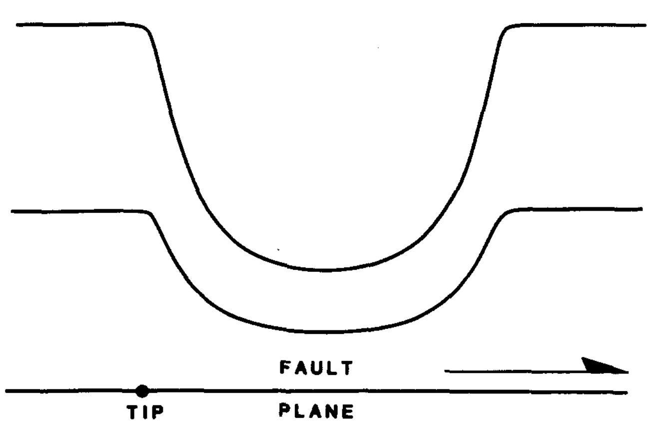

Fig. 2. Model of an extensional fault tip in which the heave increases smoothly using the function noted in equation (2). Area is conserved, and shear strains on the limbs of the hangingwall syncline increase upwards away from the fault.

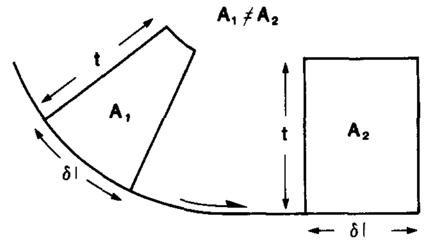

Fig. 3. The slip-line model of Williams & Vann (1987). Area is not conserved if segments of rock move over arcs of fault of different radii.

and perpendicular as they move down the fault. However, as Fig. 3 shows, if the lines remain fixed in length then area is not conserved. As in the modified Chevron construction, we may allow local area conservation if lines change in length. Consider a thin wedge-shaped portion of hangingwall (Fig. 4). Suppose a point is at a distance t from the fault where the fault's radius is R. Then the area between this point and the fault is

$$\begin{aligned}\delta A &= \tfrac{1}{2}R^2\,\delta\theta - \tfrac{1}{2}(R-t)^2\,\delta\theta \\ &= [\tfrac{1}{2}R - \tfrac{1}{2}(R-t)^2/R]\,\delta l = (t - t^2/2R)\,\delta l,\end{aligned}$$

where δl is the (fixed) length of a line adjacent and parallel to the fault. If δA and δl are fixed then t is predicted to change, as the radius of curvature of the fault changes according to

$$K = t - t^2/2R,$$

where K is constant for a given material point. Thus t attains its minimum value on planar portions of the fault, where $t_p = K$. We may also write

$$t = R - \sqrt{R^2 - 2Rt_p}.$$

The effect of this is to cause slip lines to converge on the fault plane (Fig. 5). This area-correct version of the slip-line model can be extended to include the fault slip/propagation model, in which the distance t changes not only due to changing fault curvature but also due to fault-parallel strains (δl changes).

The important feature of this argument is that it illustrates that the radius of curvature of the fault must be large for the method to work. If any point in the hangingwall lies at or beyond the centre of curvature then infinite strains are predicted. This seriously limits the usefulness of this method, and it is inapplicable if the fault shows discontinuous dip changes and the local radius of curvature is zero.

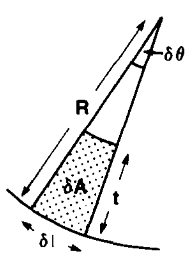

Fig. 4. Quantities used to describe hangingwall fault segments in the amended version of the slip-line model (see text).

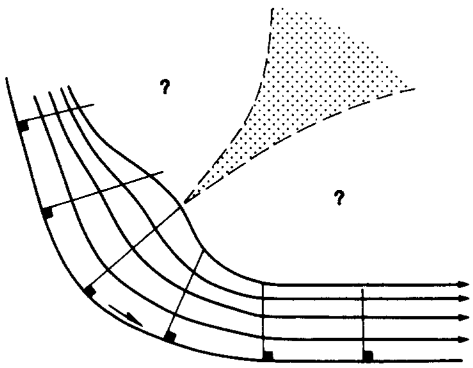

Fig. 5. In the area-conserved slip-line model, material paths (arrowed lines) converge on straighter portions of the fault. Paths for material points lying in the stippled area (which is bounded by the local centres of curvature of the fault) are undefined. Points in regions marked ? will move into or out of the stippled region, so their complete trajectories are not defined. Note the large strains away from the fault surface.

DISCUSSION

I have presented models for deformation above faults which conserve area. This does not mean that area changes cannot occur: area decreases may occur by compaction, pressure solution, etc., and area increases by dilational brecciation. The models given by Williams & Vann (1987) can be used to simulate extensional faults whose displacement decreases downwards, in which case the predicted area decrease could be assigned to compactional effects. The following comments apply to this scenario.

(a) The models could not be applied to rocks consolidated before faulting.

(b) If compaction is to be included in modelling of sediment deformation, it should be treated as a process which may accompany faulting or may be independent of it (e.g. White *et al.* 1986). It is artificial to force a complete dependence of compaction on faulting, as the modified Chevron and slip-line models would do.

(c) In other cases of changing fault slip (e.g. Fig. 3), area *increases* are predicted which would imply widespread and large dilational strains, for which there is little evidence in deformed basin sequences.

The area-conserved versions of deformation models proposed here retain the useful features of the modified Chevron and slip-line models, whilst bringing them into line with others which conserve area such as the flexural-slip model used in thrust belts, the vertical shear (Chevron) model (Verrall 1981), and the inclined shear model (White *et al.* 1986).

In summary, there are three features of the modified Chevron and slip-line models which should be appreciated before they are applied.

(1) In the form presented by Williams & Vann (1987), they both predict area increases and decreases in the hangingwall.

(2) The models can be corrected to conserve area, as demonstrated here.

(3) Both models predict strains which increase indefinitely away from the fault (Figs. 2 and 5). The slip-line model breaks down completely at distances at or beyond the locus of local centres of curvature of the fault surface. These effects limit the usefulness of the models in geological cross-sections. They are due partly to the guiding notion that slip on faults determines the deformation style in their hangingwalls, whilst in reality the opposite is probably true.

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